Note by Prof. W. W. Johnson.— Problem 85, in Analyst for September, is identical with No. 85 of the Exercises in Chauvenet's Geometry, the statement being made in the form which Prof. Scheffer gives in the November No. Prof. Scheffer is undoubtedly right in stating that the problem cannot be solved by the rule and compass only. The following will show that it has theoretically six solutions, and in some cases they are all real solutions.

Assume the given point as origin and the line joining it with the centre of one of the circles as initial line, the polar equation of that circle is

$$r^2 - 2ar\cos\theta + a^2 = b^2,$$

or

$$r = a\cos\theta \pm \sqrt{(b^2 - a^2\sin^2\theta)}.$$

Construct a curve by increasing and diminishing each radius vector by d, the length of the fourth side of the quadrilateral or line to be placed between the circumferences. The equation to this curve is therefore

$$r = a\cos\theta \pm d \pm \sqrt{(b^2 - a^2\sin^2\theta)},$$

or

$$r^2 - 2ar\cos\theta \mp 2d(r - a\cos\theta) + a^2 + d^2 - b^2 = 0,$$

or, partially introducing rectangular coordinates,

$$x^2 + y^2 - 2ax + a^2 + d^2 - b^2 = \pm 2d(r - a\cos\theta).$$

Multiplying both members by r and squaring we have

$$(x^2 + y^2)(x^2 + y^2 - 2ax + a^2 + d^2 - b^2)^2 = 4d^2(x^2 + y^2 - ax)^2 \,.\,.\, (1)$$

Now whenever this locus is cut by the other circle we have a solution of the problem. The equation of this last circle will be of the form

$$x^2 + y^2 + Ax + By + C = 0. \,.\,.\,.\,.\,.\,.\,.\,.\,.\, (2)$$

Substituting the value of $x^2 + y^2$ from (2) into (1), (namely the linear expression, $-Ax - By - C$) we have an equation of the third degree; which cubic combined with equation (2) will give theoretically six values of x and y. Constructing the curve (1) in the case when the distance of the pole from the circumference of the first circle is less than d, it is evident that the circle (2) may cut (1) in six real points, hence there may be six real solutions. The position of the given circles and the given point in this case is as follows — The circles being of unequal radii intersect, and the given point is within the smaller of the crescent-shaped figures formed. Let a straight line pass through the given point and revolve about it, the intercepts made on this line by each of the crescent-shaped and the lens-shaped figures will vary, and each may evidently twice become equal to the line to be inscribed provided the latter is within the proper limits.